metal coating; $v_x/(v_x)_m$, relative stream velocity component at a point of the fluid jet boundary layer; $(v_x)_m$, maximum value of the velocity component in the cross section of the fluid jet stream; $W/W_m$, relative velocity in the cross section of an air jet incident on a solid surface; $W_m$, maximum value of the velocity in the stream cross section; $P - P_0$, excess pressure in the stream; $P_0$, atmospheric pressure; $\rho_0$, air jet density; $R^*$, width of the air jet cross section at the solid surface.

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# ISOTHERMAL SLIP OF A BINARY GAS MIXTURE ALONG A SOLID SURFACE

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This paper investigates Couette flow of a binary mixture of a rarefied gas between two flat plates. An analytical expression for the rate of isothermal slip of a binary mixture of the rarefied gas along the surface is obtained.

There are a large number of references (see, e.g., [1-5]), dealing with slip of a gas along a surface. However, because of the difficulties in computing the distribution function, only slip of a Maxwellian gas and of a gas interacting according to the solid sphere law were considered in [2-5]. The gas slip was investigated by a variational method in [6] for arbitrary direction of the molecules. The molecular distribution function was sought in the form of an expansion of a near-Maxwellian distribution function with the mean-mass mixture velocity. The validity of this from the physical point of view must be subject to doubt. In fact, far from the surface, in the Navier–Stokes flow region, the difference between the partial and the mean-mass velocities is a small quantity of order $K_n = (\lambda/L) \ll 1$ ($\lambda$ is the mean free path, and L is a characteristic hydrodynamic length). However, near the surface, where $K_n \sim 1$, a state of the gas mixture can exist where the difference between the partial and the mean-mass velocities is on the order of the mean-mass velocity [7].

A modified semi-three-dimensional method of moments, developed by the present author in [8], has been used to investigate Couette flow of a binary mixture of a rarefied gas between two parallel plates. An expression has been obtained for the slip velocity with arbitrary law of interaction of the molecules with each other and with the surface. It should be noted that because of the transfer of viscous momentum across the Knudsen layer by diffusion velocities, the slip coefficient is reduced by 20% for a specific ratio between the parameters of the mixture components.

We now consider the problem of Couette flow of a binary mixture of gases between two surfaces a distance 2L apart. The upper surface moves with velocity $u_y$ in the direction of

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the y axis, and the lower surface moves with velocity $u_y$. The x axis is perpendicular to the plate. We seek a solution of the Boltzmann equation in the form of a finite expansion in Hermite–Chebyshev polynomials. We have the following system of equations [8] for the coefficients of this expansion:

$$\nabla_x \sigma^+_{\alpha xy} = \frac{n_\alpha n_\beta T}{n D_{\alpha\beta}} (u_\alpha - u_\beta) + \sum_\beta \left( \frac{g_{\alpha\beta}}{\lambda^-_\beta} + \delta_{\alpha\beta} \frac{g_\alpha}{\lambda^-_\alpha} \right) \sigma^-_{\beta xy},$$

$$\nabla_x \sigma^-_{\alpha xy} = (\mu^-_\alpha/(\lambda^-_\alpha)^2) u^-_{\alpha y}, \tag{1}$$

$$\nabla_x u^+_{\alpha y} = \sum_\beta \frac{a_{\alpha\beta}}{y_\alpha y_\beta} \sigma^+_{\beta xy} + \sum_\beta \left( \frac{h_{\alpha\beta}}{\lambda^-_\beta} - \delta_{\alpha\beta} \frac{h_\alpha}{\lambda^-_\alpha} \right) u^-_{\beta y},$$

$$\nabla_x u^-_{\alpha y} = (1/\mu^-_\alpha) \sigma^-_{\alpha xy},$$

where

$$u^\pm_{\alpha y} = \frac{1}{2} (u_{\alpha y} \pm u_{-\alpha y}); \quad \sigma^\pm_{\alpha xy} = \frac{1}{2} (\sigma_{\alpha xy} \pm \sigma_{-\alpha xy});$$

$$g_\alpha = 4 \frac{\lambda^-_\alpha}{v_{T\alpha}} \sum_\beta \{ M^{3/2}_\alpha \omega_{\alpha\beta} [t^2_y t_x] + M^{1/2}_\alpha M_\beta \omega_{\alpha\beta} [t_x W_y W'_y] -$$
$$- M_\alpha M^{1/2}_\beta \omega_{\alpha\beta} [t^2_y W_x] - M^{3/2}_\beta \omega_{\alpha\beta} [W_x W_y W'_y] \};$$

$$g_{\alpha\beta} = 4 \frac{\lambda^-_\beta}{v_{T\beta}} \frac{n_\alpha}{n_\beta} M_\alpha \{ M^{1/2}_\beta \omega_{\alpha\beta} [t_x W_y (W_y - W'_y)] + M^{1/2}_\alpha \omega_{\alpha\beta} [W_x W_y (W_y - W'_y)] \};$$

$$h_{\alpha\beta} = 4 \frac{\lambda^-_\beta}{v_{T\beta}} M^{1/2}_\alpha M^{1/2}_\beta \{ M^{1/2}_\alpha \omega_{\alpha\beta} [t_x W_y (W_y - W'_y)] - M^{1/2}_\beta \omega_{\alpha\beta} [W_y (W_y - W'_y)] \};$$

$$h_\alpha = 4 \frac{\lambda^-_\alpha}{v_{T\alpha}} \sum_\beta \{ -M^{1/2}_\alpha \omega_{\alpha\beta} [t_x W_y W'_y] + M^{1/2}_\beta \omega_{\alpha\beta} [W_y (W_x W_y - W'_x W'_y)] \}.$$

Here $\omega_{\alpha\beta}[\varphi]$ is a function defined by the formula

$$\omega_{\alpha\beta} [\varphi (\vec{t}, \vec{W}, \vec{W}')] = \frac{n_\beta}{(\pi v_{T\alpha} v_{T\beta})^3} \iint d\vec{t} d\vec{W} d\Omega \sigma_{\alpha\beta} (W, \vartheta) \times$$
$$\times W \operatorname{sign} v_{\alpha x} \exp \left\{ - \frac{t^2}{v_0^2} - \frac{W^2}{v^2} \right\} \varphi \left( \frac{\vec{t}}{v_0}, \frac{\vec{W}}{v}, \frac{\vec{W}'}{v} \right),$$

where $\sigma_{\alpha\beta}(W, \vartheta)$ is the differential scattering cross section; $d\Omega$ is an element of solid angle; and

$$\vec{v}_{\alpha x} = \vec{t}_x - M_\beta \vec{W}_x; \quad v_0^2 = v^2_{T\alpha} v^2_{T\beta} / v^2; \quad v^2 = v^2_{T\alpha} + v^2_{T\beta};$$
$$v^2_{T\alpha} = 2T/m_\alpha; \quad M_\alpha = m_\alpha / (m_\alpha + m_\beta).$$

The values of $\lambda^-_\alpha$ were given in [8], and $u_{\pm\alpha}$ and $\sigma_{\pm\alpha xy}$ are the expansion coefficients for the distribution function of reflected and incident molecules [7].

We shall assume that the binary gas mixture is in a steady state between the plates and that the mean-mass gas velocity has a linear shape with gradient $(\nabla_x u_0)_0$. Then, using the law of partial reflection of molecules from surfaces, from Eq. (1) we have the following expression for the slip velocity of the gas mixture:

$$u = K w \mu (\nabla_x u_0)_0, \tag{2}$$

where

$$K = \left\{ 1 - \frac{n D_{\alpha\beta}}{n_\alpha n_\beta T} \frac{1}{\lambda} \frac{\alpha}{\delta_1 + \Delta} \left( w^{-1}_\alpha \frac{\mu_\beta}{\mu} - w^{-1}_\beta \frac{\mu_\alpha}{\mu} \right) \right\};$$

$$w^{-1} = \sum_\beta w^{-1}_\beta; \quad w^{-1}_\alpha = \sum_\alpha w^{-1}_{\alpha\beta}; \quad \delta_1 = \left( 1 + \exp \left\{ - \frac{2L}{\lambda} \right\} \right) \Big/ \left( 1 - \exp \left\{ - \frac{2L}{\lambda} \right\} \right);$$

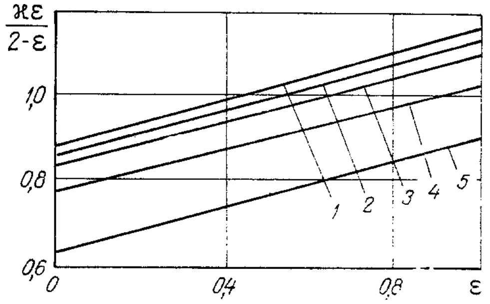


Fig. 1. The quantity $\varkappa\varepsilon/(2-\varepsilon)$ as a function of the accommodation coefficient $\varepsilon$ for various values of $K_n = 2L/\lambda$: 1) $2L/\lambda_- = \infty$; 2) 4; 3) 3; 4) 2; 5) 1.

$$\alpha = -\left[w_\alpha^{-1}\frac{\rho_\beta}{\rho} - w_\beta^{-1}\frac{\rho_\alpha}{\rho}\right]; \quad \Delta = \frac{nD_{\alpha\beta}}{n_\alpha n_\beta T}\frac{w}{\lambda}\left(w_{\alpha\alpha}^{-1}w_{\beta\beta}^{-1} - w_{\alpha\beta}^{-1}w_{\beta\alpha}^{-1}\right),$$

$$w_{\alpha\alpha}^{-1} = \frac{1}{\Delta_d}\left(\frac{\mu_\alpha^-}{\lambda_\alpha^-}c_{\alpha\alpha}d_{\alpha\alpha} - \frac{\mu_\beta^-}{\lambda_\beta^-}c_{\alpha\beta}d_{\beta\alpha}\right); \quad w_{\alpha\beta}^{-1} = \frac{1}{\Delta_d}\left(\frac{\mu_\beta^-}{\lambda_\beta^-}c_{\alpha\beta}d_{\alpha\alpha} - \right.$$

$$\left. - \frac{\mu_\alpha^-}{\lambda_\alpha^-}c_{\alpha\alpha}d_{\alpha\beta}\right); \quad \Delta_d = d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}d_{\beta\alpha}; \quad c_{\alpha\alpha} = -(g_{\alpha\alpha} + g_\alpha)\delta_\alpha^-;$$

$$c_{\alpha\beta} = -g_{\alpha\beta}\delta_\beta^+, \quad d_{\alpha\alpha} = \delta_{\varepsilon\alpha}^{-1}\left\{\left(1 + \frac{m_\alpha^0}{\sqrt{\pi}}\delta_{\varepsilon\alpha}\right) - \delta_{\varepsilon\alpha}\delta_\alpha^-(h_{\alpha\alpha} - h_\alpha)\right\};$$

$$d_{\alpha\beta} = -h_{\alpha\beta}\delta_\alpha^-; \quad m_\alpha^0 = \frac{\mu_\alpha^-}{\lambda_\alpha^-}\frac{v_{T\alpha}}{n_\alpha T}; \quad \delta_{\varepsilon\alpha} = \frac{\varepsilon_\alpha}{2 - \varepsilon_\alpha};$$

$$\delta_\alpha^\pm = 1 \pm \exp\left\{-\frac{2L}{\lambda_\alpha^-}\right\}.$$

Here $\varepsilon_\alpha$ is the partial accommodation coefficient; and $\lambda$, $\lambda_\alpha^-$, $D_{\alpha\beta}$, $\mu$, $\mu_\alpha^-$ are as given in [8].

In the case of a simple gas, for $L \gg \lambda$, the coefficient K = 1, and Eq. (2) transforms into the expression for slip of a single-component gas along a surface [7]. Figure 1 shows the coefficient of isothermal slip of a simple gas as a function of the accommodation coefficient and agrees with the slip coefficient obtained in [9]. For a binary mixture, the coefficient K differs from 1 because there is a second term in the expression. This term is due to viscous momentum transfer across the Knudsen layer, from the diffusion rate, and goes to zero for a simple gas. In the case of a binary mixture of a Maxwellian gas far from the surface, the factor K can be represented in the form

$$K = 1 + \frac{\delta_\varepsilon}{\sqrt{2}}\frac{(y_\alpha y_\beta)^{3/2}}{d\sqrt{\pi}}\left\{\frac{2}{(2-\varepsilon)d}\frac{\varepsilon_\alpha - \varepsilon_\beta}{\varepsilon_\alpha + \varepsilon_\beta} + \left(1 + \delta_\varepsilon\frac{1}{d\sqrt{\pi}}\right)x_\alpha + \right.$$

$$\left. + \frac{4}{3}z_\alpha\right\}\left\{\left(1 - \delta_\varepsilon\frac{1}{2\sqrt{\pi}d}\right)x_\alpha - \frac{3}{4}z_\alpha\right\}; \quad x_\alpha = \frac{m_\alpha - m_\beta}{m_\alpha + m_\beta};$$

$$z_\alpha = 2\frac{\sigma_\alpha - \sigma_\beta}{\sigma_\alpha + \sigma_\beta}; \quad |\varepsilon_\alpha - \varepsilon_\beta| \ll (\varepsilon_\alpha + \varepsilon_\beta)/2 = \varepsilon, \quad d_{\alpha\alpha} = d_{\beta\beta} = d.$$

If we substitute these expressions into Eq. (2), it turns out that the contribution to the slip velocity due to viscous momentum transfer across the Knudsen layer by diffusion rates is given by a quadratic form in $x_\alpha$, $z_\alpha$, $(\varepsilon_\alpha - \varepsilon_\beta)$. For example, for $m_\alpha \gg m_\beta$, $\sigma_\alpha \gg \sigma_\beta$, $\varepsilon_\alpha = \varepsilon_\beta = 1$, $y_\alpha = y_\beta = 1/2$ because of diffusion transfer the slip coefficient is reduced by 20%.

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